Bayesian spatio-temporal methods for small-area estimation of HIV indicators

Adam Howes

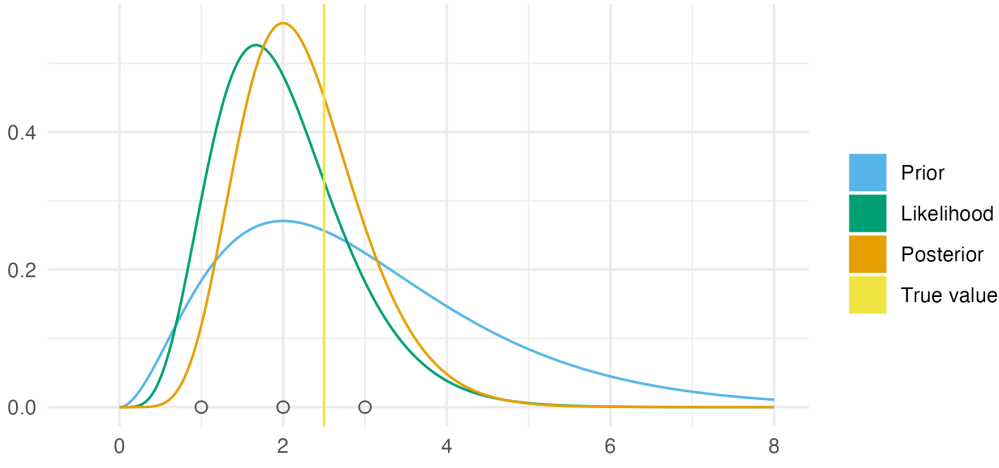
Imperial College London

March 2023

Bayesian

Use probability distributions for all unknowns

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$
$$p(\phi \mid \mathbf{y}) \propto p(\mathbf{y} \mid \phi) \times p(\phi)$$

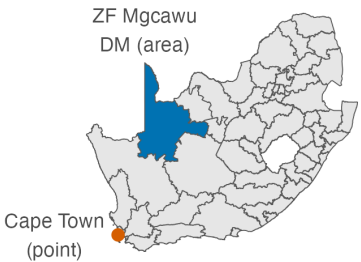

0.4
0.2
0.0
0
2
4
6
8
Prior
Likelihood
Posterior
True value

Spatio-temporal

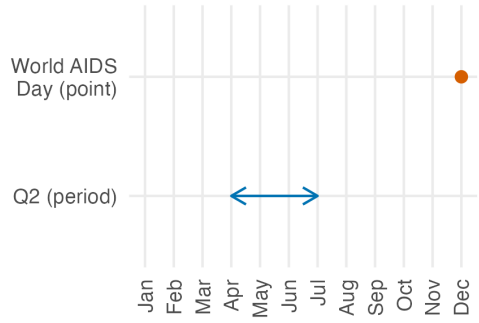
Observed data has spatial and temporal location

$y_{s,t}$ where s is space and t is time

A


ZF Mgcawu
DM (area)
Cape Town
(point)

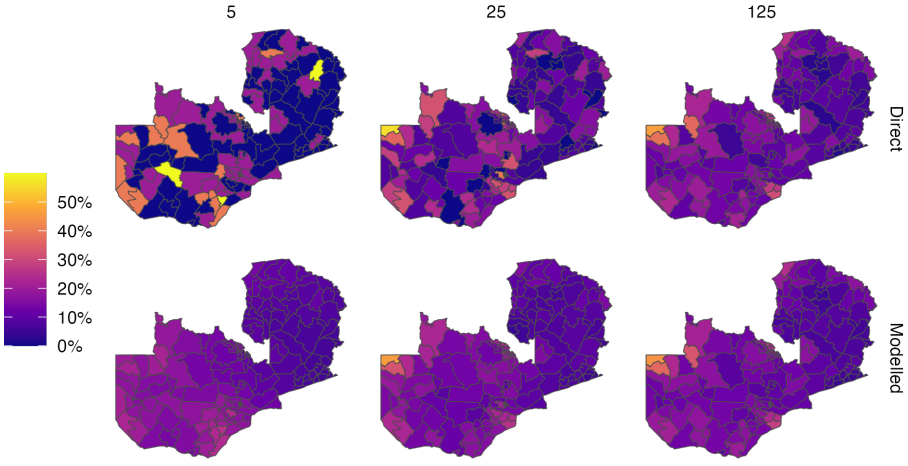
B

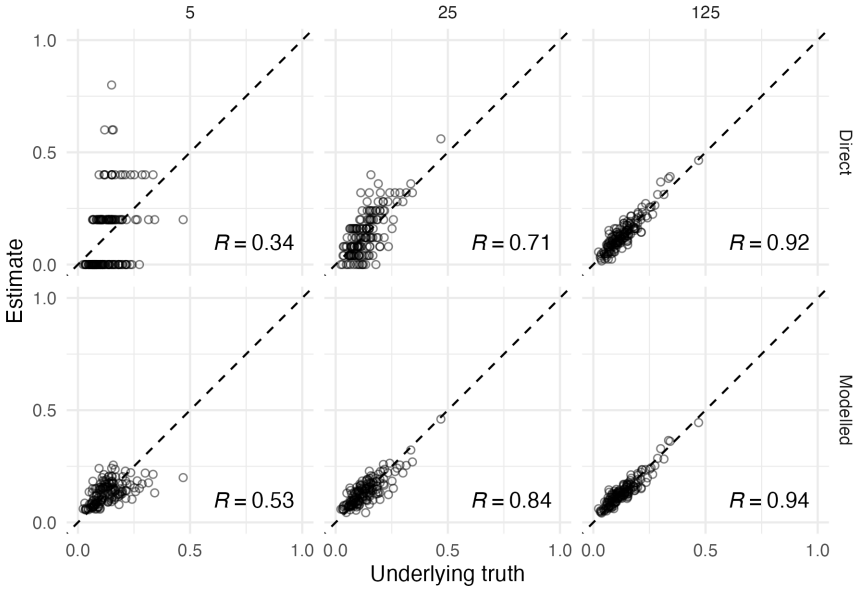

World AIDS
Day (point)
Q2 (period)
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec

Small-area estimation

Sample size for demographic subgroups too low for precise direct estimates

By space s and time t together with other factors like age and sex


5
25
125
Direct
Modelled
50%
40%
30%
20%
10%
0%

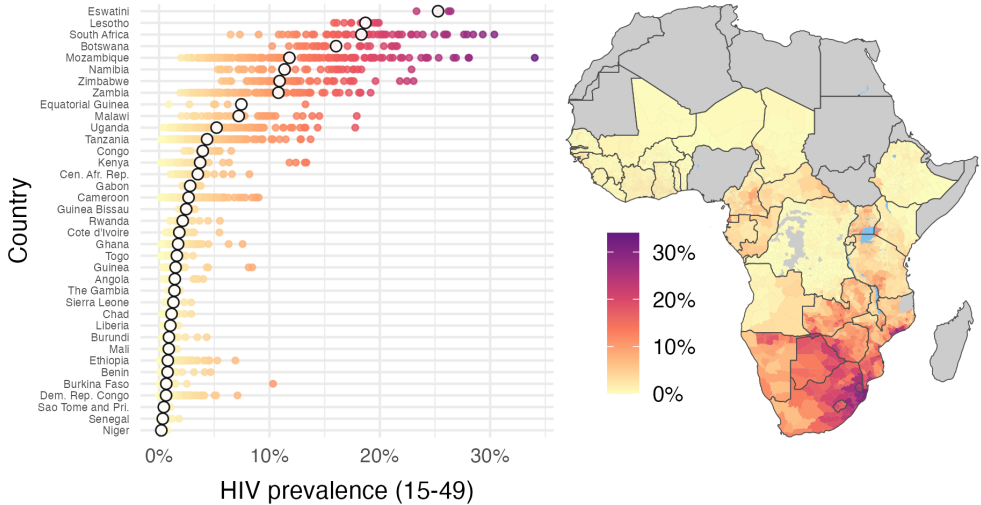

5
25
125
Direct
Modelled
Estimate
Underlying truth
R = 0.34
R = 0.71
R = 0.92
R = 0.53
R = 0.84
R = 0.94
0.0
0.5
1.0

Toy example! But the same principle applies to real models

e.g. Naomi[1] (Eaton et al. 2021; Esra et al. 2024)

> *"Now I understand how Naomi works!"*
>
> *— Anonymous, fictionalised, workshop participant*

[1] See C.4 Simplified Naomi model description for details

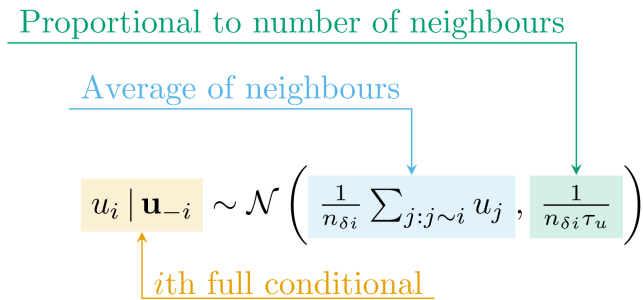


Source: UNAIDS Naomi model estimates, 2023

Nearby things tend to be similar

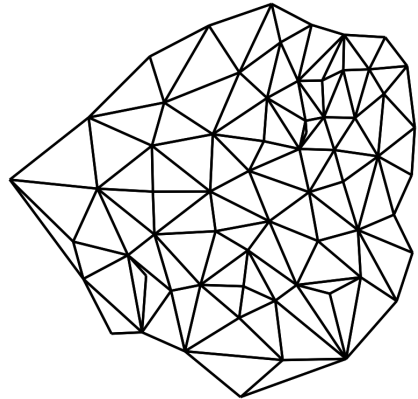
Suppose prior correlation structure between observations!

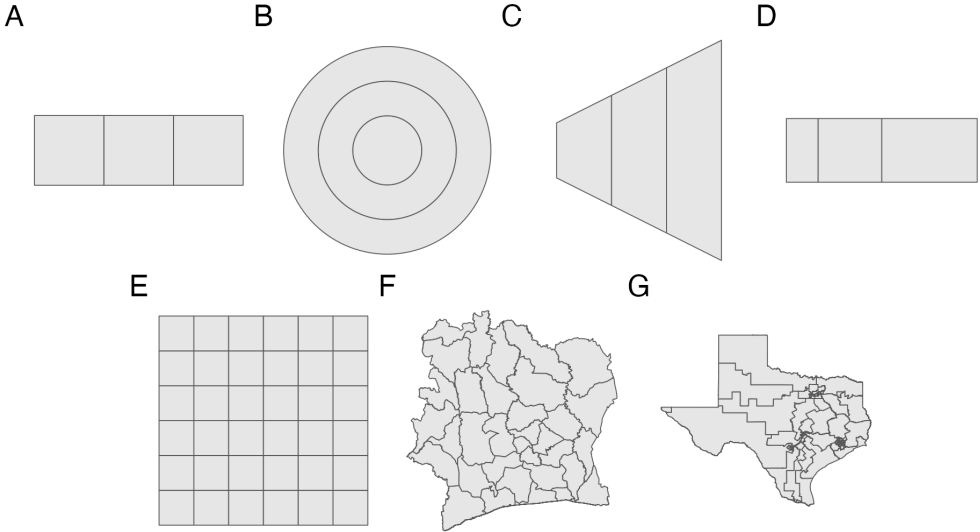
Gaussian Markov random field model of Besag, York, and Mollié (1991)



A

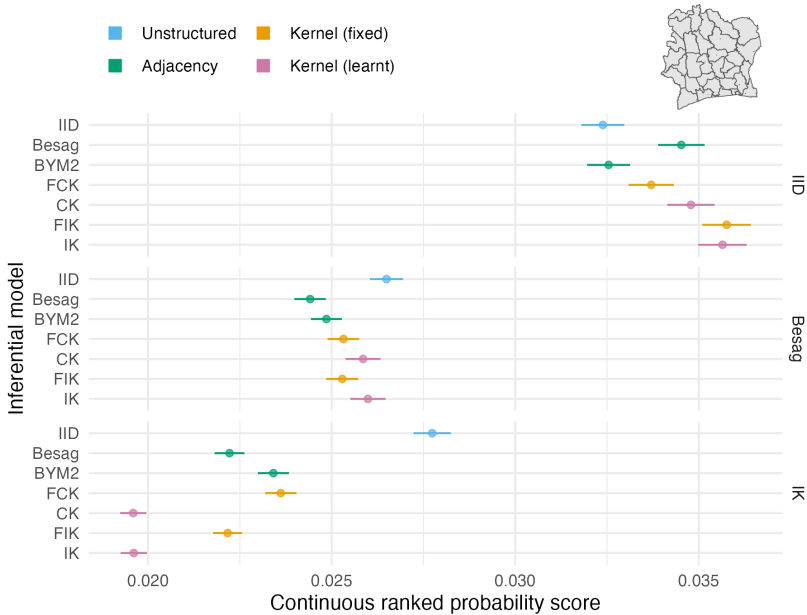
B




A
B
C
D
E
F
G

Measure forecast performance using strictly proper scoring rules

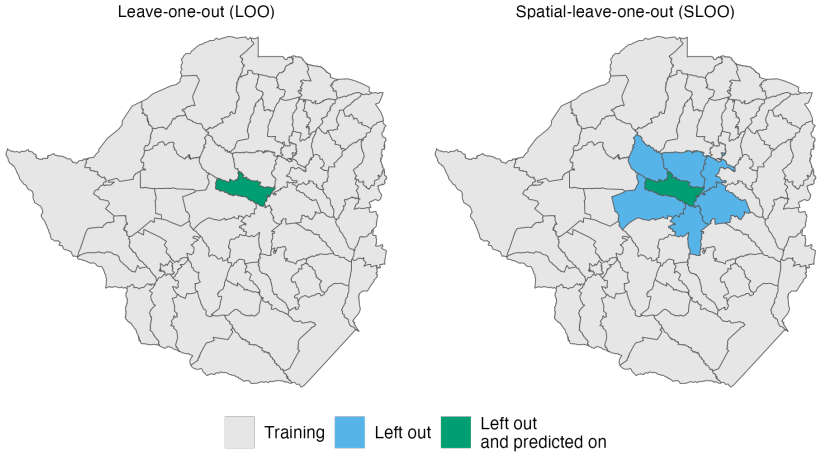
Evaluates the whole distribution and does not incentivize dishonesty (Gneiting and Raftery 2007)

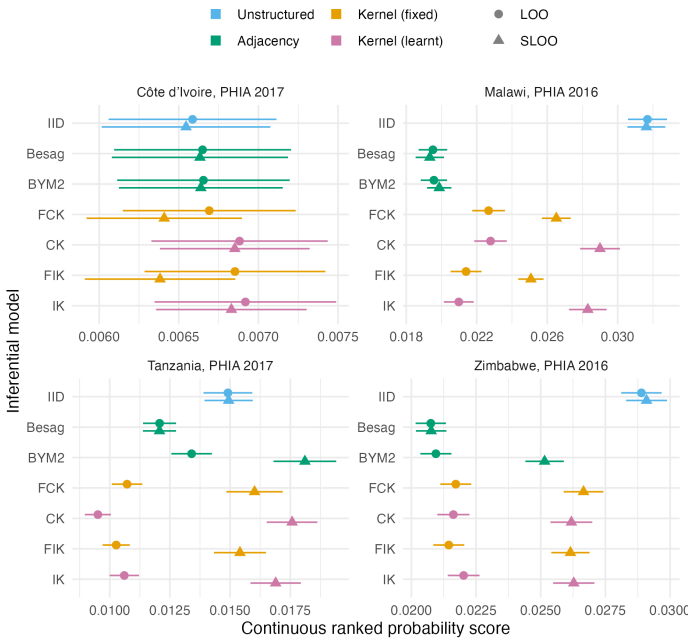

Unstructured
Kernel (fixed)
Adjacency
Kernel (learnt)
Inferential model
IID
Besag
BYM2
FCK
CK
FIK
IK
IID
Besag
IK
0.020
0.025
0.030
0.035
Continuous ranked probability score

Model comparison on real data is more challenging than with simulated data

Information criteria: adjust within-sample scoring rule performance
Cross-validation: partition data to estimate out-of-sample scoring rule performance

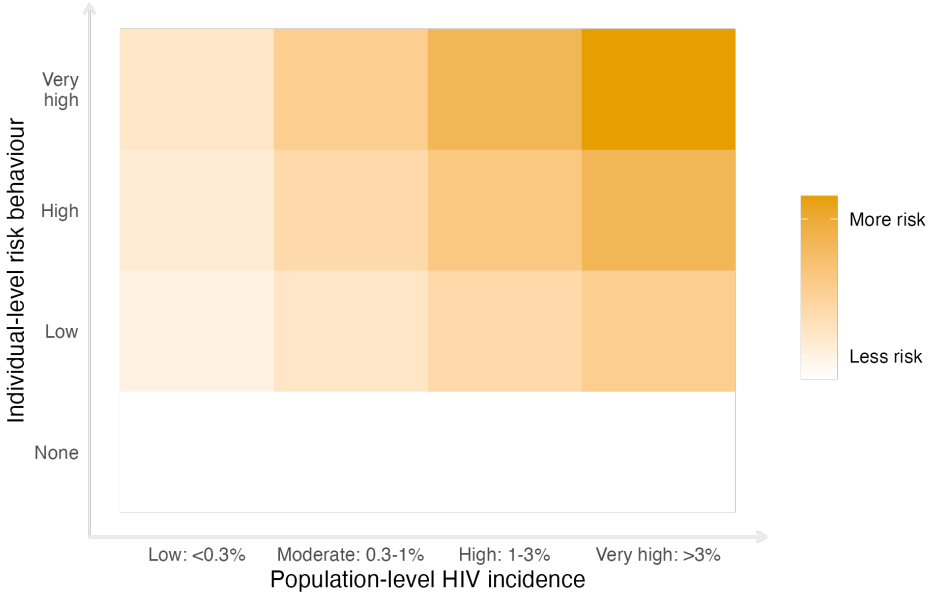
I'd recommend Vehtari (2020) for an overview

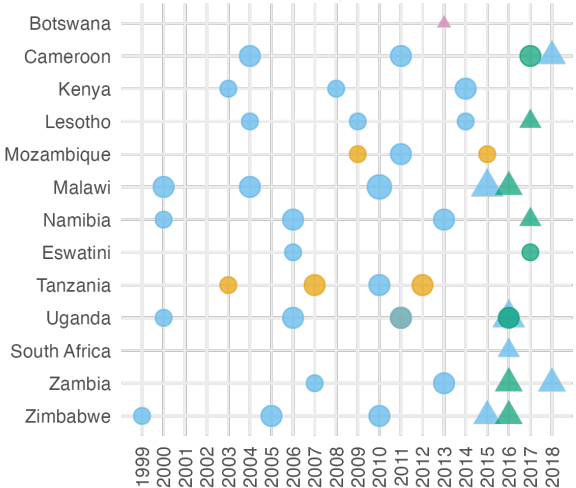

Leave-one-out (LOO)
Spatial-leave-one-out (SLOO)
Training
Left out
Left out
and predicted on


Unstructured
Adjacency
Kernel (fixed)
Kernel (learnt)
LOO
SLOO
Côte d'Ivoire, PHIA 2017
Malawi, PHIA 2016
Tanzania, PHIA 2017
Zimbabwe, PHIA 2016
IID
Besag
BYM2
FCK
CK
FIK
IK
Inferential model
0.0060
0.0065
0.0070
0.0075
0.018
0.022
0.026
0.030
0.0100
0.0125
0.0150
0.0175
0.0200
0.0225
0.0250
0.0275
0.0300
Continuous ranked probability score

GLOBAL AIDS STRATEGY 2021-2026

END INEQUALITIES.
END AIDS.


Individual-level risk behaviour
Very high
High
Low
None
Low: <0.3%
Moderate: 0.3-1%
High: 1-3%
Very high: >3%
Population-level HIV incidence
More risk
Less risk

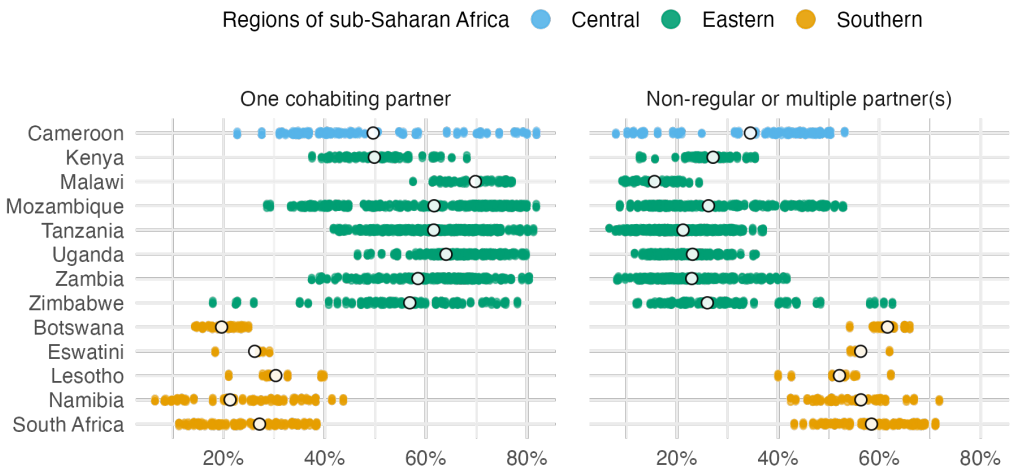

Botswana
Cameroon
Kenya
Lesotho
Mozambique
Malawi
Namibia
Eswatini
Tanzania
Uganda
South Africa
Zambia
Zimbabwe
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
Survey type
AIS
DHS
BAIS
PHIA
Sample size
<2.5k
5k-10k
2.5k-5k
10k-15k
Does the survey include a specific question about transactional sex?
Yes
No

Use the multinomial-Poisson transformation of Baker (1994)

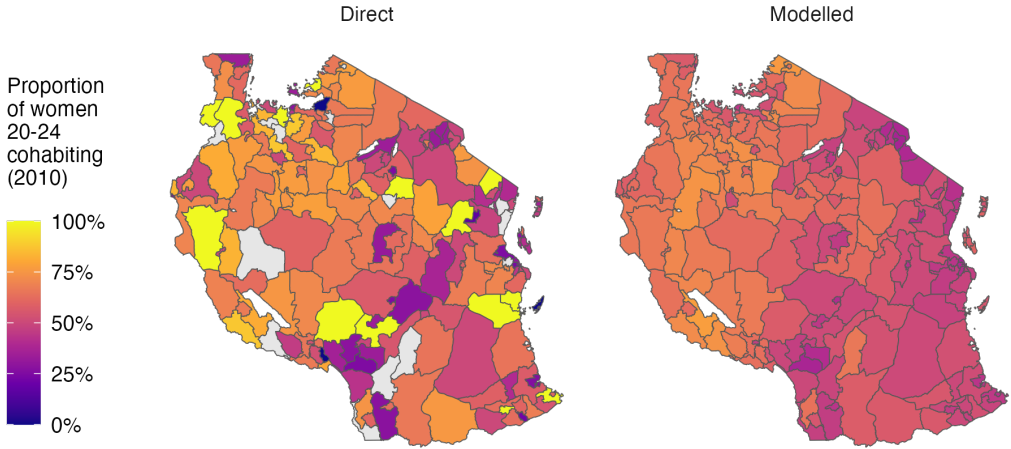
A multinomial logistic regression on $\mathbf{y} = (y_1, \ldots, y_K)$ can be expressed as a Poisson regression

$$y_k \sim \text{Poisson}(\lambda_k)$$

with observation-specific random effects with recover the sample size $m = \sum_k y_k$. See blog post (Howes 2023)



Not sexually active (not shown) + one cohabiting partner + non-regular or multiple partner(s) + FSW (not shown) = 100%


Direct
Modelled
Proportion of women 20-24 cohabiting (2010)
100%
75%
50%
25%
0%

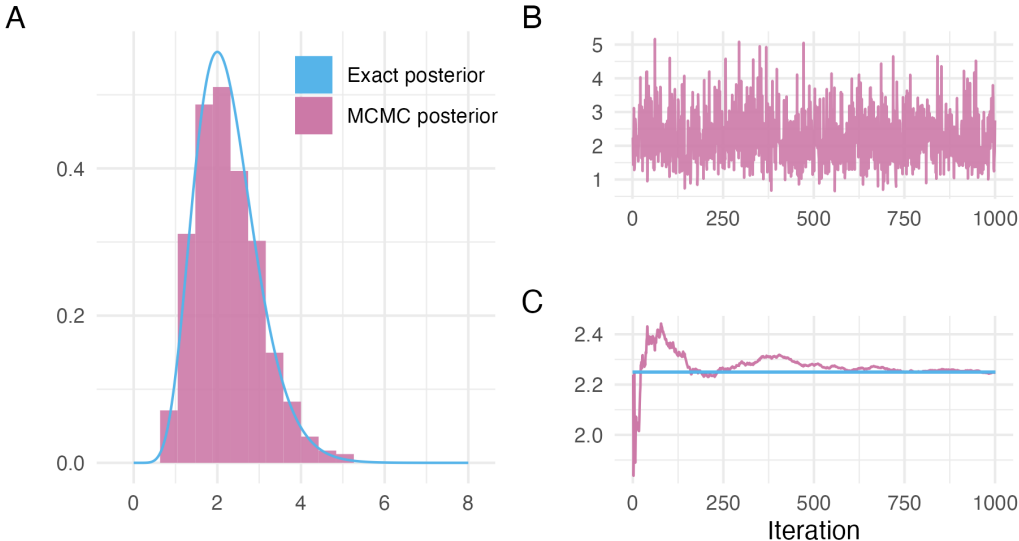
Since extended to include 1) males, 2) additional countries

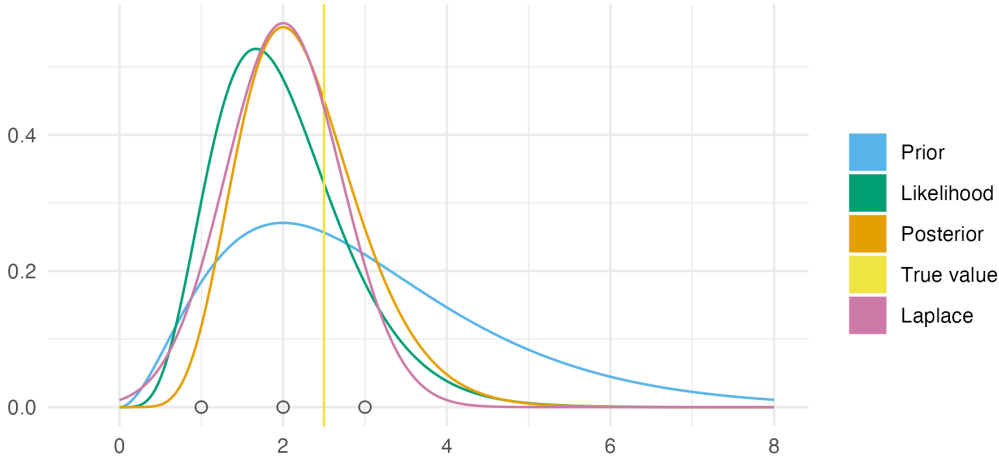
See the sub-national HIV estimates in priority populations UNAIDS tool at hivtools.unaids.org/shipp/

Bayesian computation amounts mostly to solving challenging integrals

$$p(\mathbf{y}) = \int p(\mathbf{y}, \phi) \mathrm{d}\phi$$

Methods can broadly be divided into “stochastic” and “deterministic”


A
Exact posterior
MCMC posterior
0.4
0.2
0.0
0
2
4
6
8
B
5
4
3
2
1
0
250
500
750
1000
C
2.4
2.2
2.0
0
250
500
750
1000
Iteration


0.4
0.2
0.0
0
2
4
6
8
Prior
Likelihood
Posterior
True value
Laplace

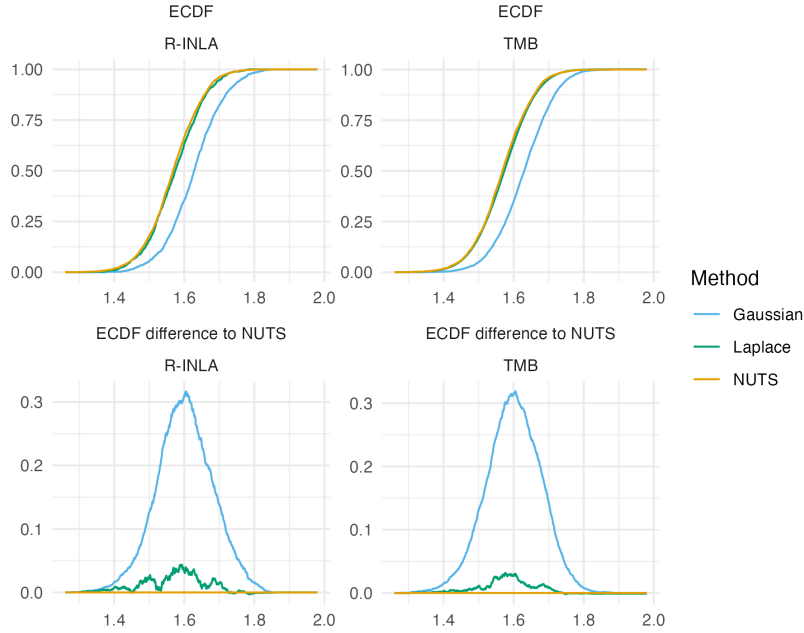
Adam Howes
@adamhowes

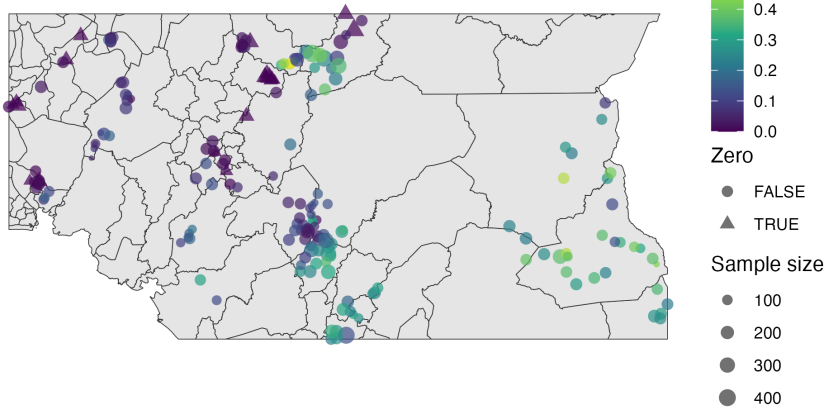
You're right that INLA and R-INLA are often synonymous. From an applied perspective, models which (for one reason or another) cannot be written using R-INLA's formula interface are typically outside the remit of the INLA inference algorithm.

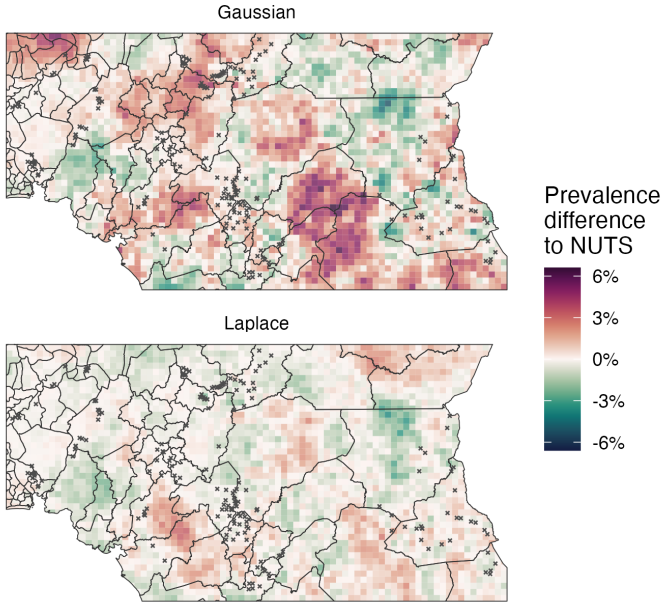
6:42 PM · Feb 28, 2024 · **155** Views

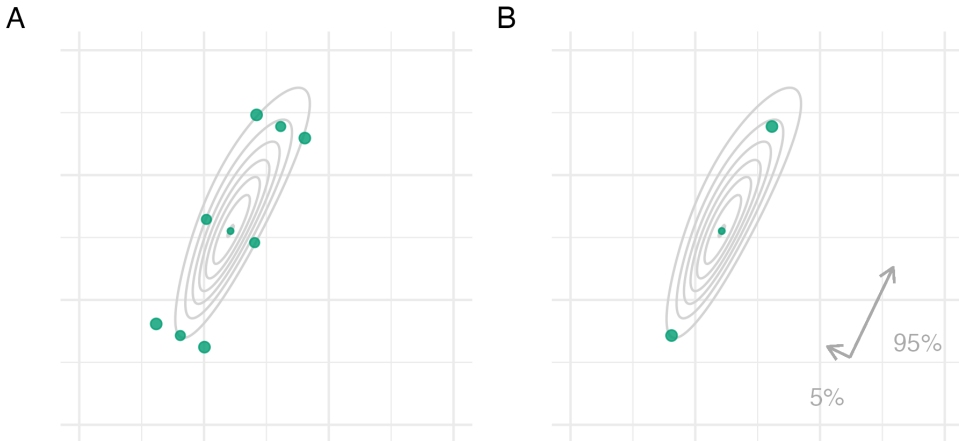
View post engagements


Treatment
1
0
0
25
50
75
100
Number of seizures


ECDF
R-INLA
ECDF
TMB
1.00
0.75
0.50
0.25
0.00
1.4
1.6
1.8
2.0
ECDF difference to NUTS
R-INLA
ECDF difference to NUTS
TMB
0.3
0.2
0.1
0.0
Method
Gaussian
Laplace
NUTS

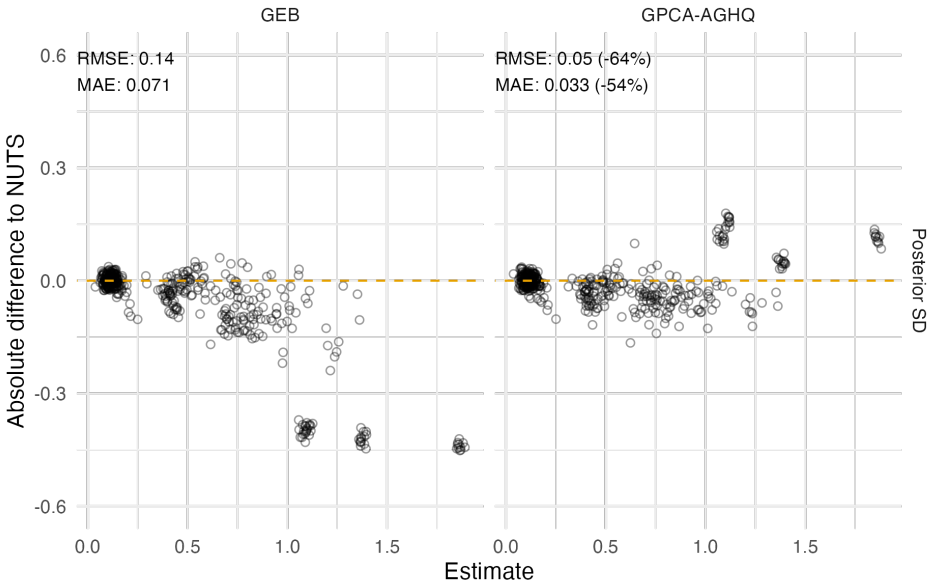

Prevalence
0.5
0.4
0.3
0.2
0.1
0.0
Zero
FALSE
TRUE
Sample size
100
200
300
400

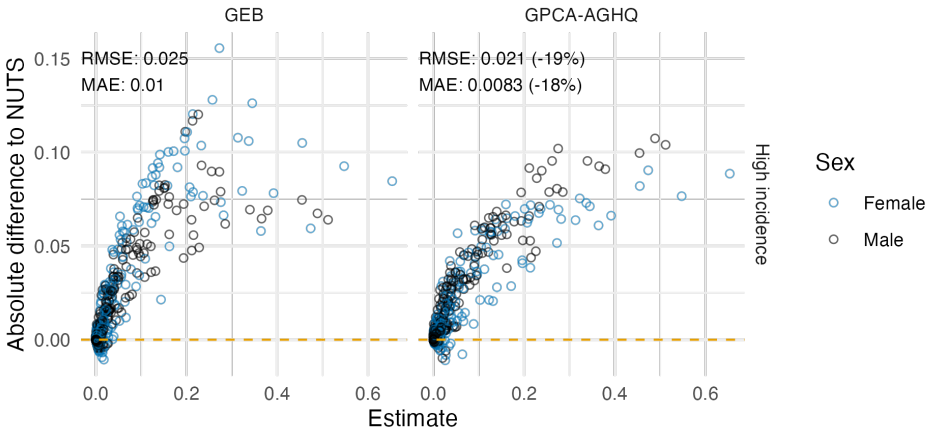

Gaussian
Laplace
Prevalence difference to NUTS
6%
3%
0%
-3%
-6%



AGHQ (spectral)

PCA-AGHQ


GEB
GPCA-AGHQ
RMSE: 0.14
MAE: 0.071
RMSE: 0.05 (-64%)
MAE: 0.033 (-54%)
Absolute difference to NUTS
Posterior SD
Estimate
0.6
0.3
0.0
-0.3
-0.6
0.0
0.5
1.0
1.5


GEB
GPCA-AGHQ
RMSE: 0.025
MAE: 0.01
RMSE: 0.021 (-19%)
MAE: 0.0083 (-18%)
Absolute difference to NUTS
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
Estimate
High incidence
Sex
Female
Male

Acknowledgements

Collaborator(s)	Affiliation
Jeff Eaton	Imperial, Harvard
Seth Flaxman	Oxford
Alex Stringer	Waterloo
HIV Inference Group	Imperial
Machine Learning and Global Health Network	Worldwide
StatML CDT	Imperial, Oxford

What am I up to now?

- Nowcasting food insecurity with the WFP using MRP and active learning
- Starting forecasting work with CDC soon – especially interested in disease-agnostic methods and data source integration
- Still based in London! Keep in touch

References I

Baker, Stuart G. 1994. "The multinomial-Poisson transformation." *Journal of the Royal Statistical Society: Series D (The Statistician)* 43 (4): 495–504.

Besag, Julian, Jeremy York, and Annie Mollié. 1991. "Bayesian image restoration, with two applications in spatial statistics." *Annals of the Institute of Statistical Mathematics* 43 (1): 1–20.

Eaton, Jeffrey W, Laura Dwyer-Lindgren, Steve Gutreuter, Megan O'Driscoll, Oliver Stevens, Sumali Bajaj, Rob Ashton, et al. 2021. "Naomi: a new modelling tool for estimating HIV epidemic indicators at the district level in sub-Saharan Africa." *Journal of the International AIDS Society* 24: e25788.

References II

Esra, Rachel, Mpho Mmelesi, Akeem T. Ketlogetswe, Timothy M. Wolock, Adam Howes, Tlotlo Nong, Matshelo Tina Matlhaga, Siphiwe Ratladi, Dinah Ramaabya, and Jeffrey W. Imai-Eaton. 2024. "Improved Indicators for Subnational Unmet Antiretroviral Therapy Need in the Health System: Updates to the Naomi Model in 2023." *JAIDS Journal of Acquired Immune Deficiency Syndromes* 95 (1S): e24–33. https://doi.org/10.1097/QAI.0000000000003324.

Gneiting, Tilmann, and Adrian E Raftery. 2007. "Strictly proper scoring rules, prediction, and estimation." *Journal of the American Statistical Association* 102 (477): 359–78.

Howes, Adam. 2023. "Adam Howes: Estimating Risk Group Proportions: Informal Discussion." https://athowes.github.io/posts/2023-04-21-risk-group-retrospective/.

References III

Vehtari, Aki. 2020. “Cross-Validation FAQ.” *Model Selection Tutorials and Talks*.